Lecture

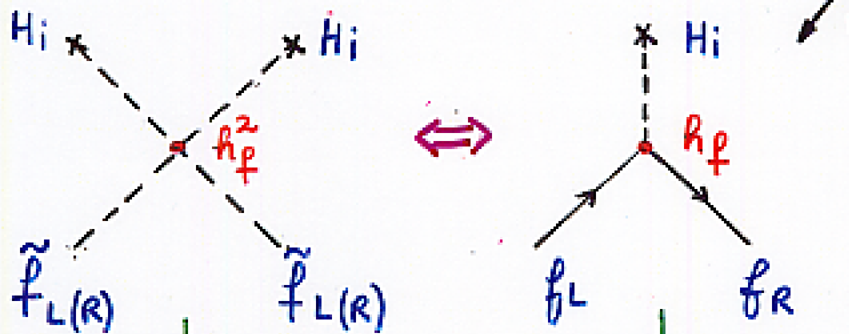
3

SUSY Breaking

- SUSY must be broken in nature, but SUSY mechanism not well understood yet.

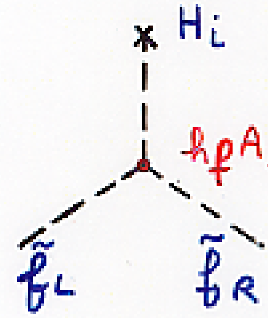
- empirically: add all possible soft SUSY terms
  $\longrightarrow$ which preserve cancellation of quad. divergence

• Scalar masses $m_{\tilde{f}_{SUSY}} \Longrightarrow m^2_{\tilde{f}} = \underbrace{m^2_f}_{SUSY} + \underbrace{m^2_{\tilde{f}_{SUSY}}}_{m_i^2}$



$$m^2_{\tilde{f}} = h_f^2 v_i^2 \qquad\qquad m_f = h_f v_i$$

• Scalar left-right mixing terms A_f:



• Gaugino mass terms $\quad M_i(\lambda_i\lambda_i + \bar{\lambda}_i\bar{\lambda}_i)$

Finite set of soft SUSY param.: $m_{\tilde{f}_{SUSY}}, A_f, M_i$

$\Rightarrow$ wide range of possible values at high energies
(dep. on SUSY mechanism)

+ Renormalization Group evolution $\longrightarrow$ determine low energy values

= Define SUSY MASS SPECTRUM

$$\mathcal{L}_{\cancel{SUSY}} = -\sum_{\text{scalars}} \underbrace{m_i^2}_{(m^2_{\tilde{f}_{SUSY}})} A_i^2 - \sum_{\text{gauginos}} M_i (\lambda_i \lambda_i + \bar{\lambda}_i \bar{\lambda}_i)$$

$$+ \left(B\, P^{(2)}[A] + \sum_k A_k\, P^{(3)}_{(k)}[A] + h.c. \right)$$

where $P^{(2)}[A] = \mu\, \varepsilon_{ij} H_1^i H_2^j$

$\mu \longrightarrow$ Higgs/Higgsino SUSY mass param.

$\underline{B} \longrightarrow$ extra, soft $\cancel{SUSY}$ term, but determined from cond. of proper EWSB (see next lecture)

$$P^{(3)}_k [A]\, A_k = \left(A_b h_b \tilde{Q}^j \tilde{D} H_1^i + A_t h_t \tilde{Q}^j \tilde{U} H_2^i + A_\tau h_\tau \tilde{L}^j \tilde{E} H_1^i \right) \varepsilon_{ij}$$

$$\tilde{Q} = \begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix} \qquad \tilde{U} = \tilde{t}^*_R \qquad \tilde{L} = \begin{pmatrix} \tilde{\nu}_\tau \\ \tilde{\tau} \end{pmatrix}_L \qquad \tilde{E} = \tilde{\tau}^*_R$$

trilinear terms, prop. to Yukawa couplings,

$\longrightarrow$ induce Left-right mixing in the squark sector when Higgs acquire v.e.v.

Mixing prop. to fermion masses

$\Rightarrow$ relevant for 3 gen. only

Soft SUSY parameters in the Stop Sector

$$V_{stop} \simeq (m_Q^2 + m_t^2)\tilde{t}_L^*\tilde{t}_L + (m_u^2 + m_t^2)\tilde{u}^*\tilde{u}$$
$$+ \left[A_t h_t H_2 \tilde{t}_L \tilde{u} - \mu h_t H_1 \tilde{t}_L \tilde{u} + h.c.\right] + \ldots$$

$$\tilde{Q} = \begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix} \qquad \tilde{u} = \tilde{t}_R^* \qquad m_q,\ m_u,\ A_t \Rightarrow \text{soft SUSY}$$

$$M_{\tilde{t}}^2 = \begin{array}{ccc} & LL & LR \\ & \left[\begin{array}{c} m_Q^2 + m_t^2 + D_{\tilde{t}_L}^2 \\ m_t\left(A_t - \mu/\tan\beta\right) \end{array}\right. & \left.\begin{array}{c} m_t\left(A_t - \mu/\tan\beta\right) \\ m_u^2 + m_t^2 + D_{\tilde{t}_R}^2 \end{array}\right] \\ & RL & RR \end{array}$$

$$D_{\tilde{t}_L}^2 = M_Z^2 \cos 2\beta \left(T_{3_t} - Q_t \sin^2\theta_W\right)$$
$$D_{\tilde{t}_R}^2 = M_Z^2 \cos 2\beta\, Q_t \sin^2\theta_W$$

$T_{3_t}, Q_t \longrightarrow$ isospin / electric charge of top.

- large mixing due to large Yukawa Coupling
- light stops, $m_{\tilde{t}} < m_t$, may occur if large mixing or $\underline{m_u^2 \lesssim 0} \longrightarrow$ Electroweak Baryogenesis
- precision measurements $\longrightarrow m_Q^2 \gg m_u^2, m_t^2$

$$m_{\tilde{t}}^2 \simeq m_u^2 + m_t^2\left(1 - \frac{X_t^2}{m_Q^2}\right) \qquad m_{\tilde{T}}^2 \simeq m_Q^2 + m_t^2\left(1 + \frac{X_t^2}{m_Q^2}\right)$$

$$X_t \equiv A_t - \mu/\tan\beta$$

MSSM:

minimal SUSY extension of the SM

$\equiv$ minimal extension of particle content

but, within the MSSM

⟶ many $\cancel{SUSY}$ scenarios possible (see next lecture)

$\Downarrow$

many different boundary conditions for $\cancel{SUSY}$ param. at the energy scale at which $\cancel{SUSY}$ is transmited to observable sector

- Generically, SUSY is broken spontaneously in some new sector of particles at high energies, when some of the components of the new (hidden) secto[r] acquire v.e.v. $\langle F \rangle \longrightarrow$ dim $[mass]^2$

interaction terms between those components and the MSSM superfields ⟶ give rise to $\mathcal{L}_{\cancel{SUSY}}$

<one can think of Messengers (w/ generic mass M) which couple hidden sector to MSSM sector $\Rightarrow m_{\cancel{SUSY}} \propto \langle F \rangle / M$

Renormalization Group evolution of $\cancel{SUSY}$ param.

$\Rightarrow \neq$ hierarchy of MSSM particle masses at low energies

$\Rightarrow \neq$ production & decay patterns

$\neq$ search strategies at colliders

• local SUPERSYMMETRY $\Longleftrightarrow$ SUPERGRAVITY

$\longrightarrow$ low energy manifestation of a Unified Theory?

Is there a hint?

$$SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5) \text{ or } SO(10) \ldots$$

$$\alpha_3\,,\ \alpha_2\,,\ \alpha_1 \longrightarrow \alpha_5\,,\ \alpha_{10}$$

Repeat the exercise we did for the SM, for the case of the MSSM:

given known values of α_{em}, $\sin^2\theta_W$, ask for

GAUGE COUPLING UNIFICATION

and check required value of $\alpha_3(M_Z)$ to see if compatible with $\alpha_3^{exp.}(M_Z)$

recall: $\dfrac{d\alpha_i}{d\ln Q^2} = b_i\, \alpha_i^2/4\pi$ $\qquad \alpha_i = g_i^2/4\pi$

$$b_1 = 3/5 \left\{ 1/6 \sum_f Y_f^2 + 1/12 \sum_s Y_s^2 \right\}$$

due to SUSY $\longrightarrow$ $b_1 = 3/5 \left\{ \frac{3}{12} \sum_f Y^2_{f_{(SM)}} + \frac{3}{12} \sum_{H_1, H_2} 2 \right\}$

hence $b_1 = 3/5 \left\{ 3/12 \left(\underset{\downarrow\, l_L}{2} + \underset{\downarrow\, l_R^c}{4} + \underset{\downarrow\, q_L}{2/3} + \underset{\downarrow\, u_R^c}{16/3} + \underset{\downarrow\, d_R^c}{4/3} \right) \underset{\downarrow\, N_g}{3} + 1 \right\} = 33/5$

The non-abelian β-function coeff. are affected also by gluinos & winos

For $SU(N) \longrightarrow \Delta b_n = 2/3\, N$

$$\Longrightarrow b_n = \overset{-11/3\, N + 2/3\, N}{-3N} + 1/3\, n_f + 1/6\, n_s$$

$$= -3N + 1/2\, N_g\, n_f^{SM} + 1/2\, n_s^{\text{"}SM\text{"}}$$

$$b_2 = -6 + 3/2 \cdot \underset{3q_L + \ell_L}{4} + 1/2\, \underset{H_1, H_2}{2} = 1 \Longrightarrow g_2 \text{ is non-asymp. free!}$$

$$b_3 = -9 + 3/2 \cdot \underset{u_L, u_R, d_L, d_R}{4} = -3 \qquad \Longrightarrow \frac{b_3 - b_2}{b_2 - b_1} = \frac{-3-1}{1 - 33/5} = 5/7$$

Unification condition yields

$$1/\alpha_3(M_Z) = \left(1 + \frac{b_3 - b_2}{b_2 - b_1}\right) \underbrace{1/\alpha_2(M_Z)}_{\sim 30} - \left(\frac{b_3 - b_2}{b_2 - b_1}\right) \underbrace{1/\alpha_1(M_Z)}_{\sim 60}$$

$$1/\alpha_3(M_Z) = 60/7 \simeq 8.5 \simeq 1/\alpha_3(M_Z)\Big|_{exp} \; ! \quad + \frac{b_i}{4} \ln\left(\frac{M_{GUT}^2}{M_Z^2}\right)$$

One can also compute M_{GUT} $\quad 1/\alpha_i(M_{GUT}) = 1/\alpha_G = 1/\alpha_i(M_Z) +$

$$M_{GUT} = \exp\left[\left(1/\alpha_1(M_Z) - 1/\alpha_2(M_Z)\right) \frac{2\pi}{b_1 - b_2}\right] M_Z \simeq \exp(33.5)\, M_Z$$

$$\underline{M_{GUT} \sim 2 \cdot 10^{16}\ \text{GeV}}$$

All done at one-loop; two-loop correc. $\rightarrow$ slight modifications

Unification of Gauge Couplings $\longrightarrow$ 2-loop RGE

- input parameters: $1/\alpha_{em}(M_Z) = 127.9$

$$\sin^2\theta_W(M_Z) = 0.2315 - 10^{-7}\left(M_t^2/\mathrm{GeV}^2 - 170^2\right) \pm 0.0003$$

$\uparrow$ Exp. pred. in $\overline{MS}$ scheme given by G_F, M_Z, α_{em}, M_t

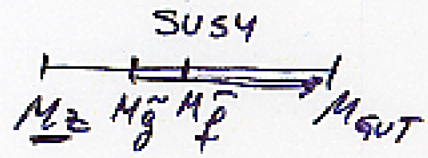
unification condition

$$1/\alpha_G = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi}\ln\left(\frac{M_{GUT}}{M_Z}\right) + \gamma_i + \Delta_i + 1/\alpha_i^{thr.}$$

γ_i : two-loop correc.

Δ_i : $\overline{DR}$ - $\overline{MS}$ correc. factor

$$1/\alpha_i^{thr} = \sum_{\eta,\, M_\eta > M_Z} \frac{b_i^\eta}{2\pi}\ln\left(M_\eta/M_Z\right)$$



$\downarrow$ SUSY threshold corrections at one-loop

* prediction for $\alpha_3(M_Z)$ as a fc. of $\alpha_1(M_Z)$, $\alpha_2(M_Z)$, α_i^{thr}

$$1/\alpha_3(M_Z) = 1/\alpha_3^{SUSY}(M_Z) + \frac{19}{28\pi}\ln\left(\frac{T_{SUSY}}{M_Z}\right)$$

Langacker, Polonski
M.C., Pokorski, Wagner

$\downarrow$ theory supersymmetric from M_{GUT} to M_Z

effect of SUSY particles with $M_\eta > M_Z$

t follows

$$\frac{1}{\alpha_3^{SUSY}(M_Z)} = \frac{b_1-b_3}{b_1-b_2}\left(\frac{1}{\alpha_2(M_Z)}+\gamma_2+\Delta_2\right)+\frac{b_2-b_3}{b_2-b_1}\left(\frac{1}{\alpha_1(M_Z)}+\gamma_1+\Delta_1\right)-\gamma_3-\Delta_3$$

while

$$\frac{19}{28\pi}\ln\left(\frac{T_{SUSY}}{M_Z}\right) = \left(\frac{b_1-b_3}{b_1-b_2}\right)\frac{1}{\alpha_2^{thr}} + \left(\frac{b_2-b_3}{b_2-b_1}\right)\frac{1}{\alpha_1^{thr}} - \frac{1}{\alpha_3^{thr}}$$

the threshold corrections to $\alpha_3(M_Z)$ (same for $\sin^2\theta_W$)
can be described by a single effective scale T_{SUSY}

$$T_{SUSY} = m_{\tilde{H}}\left(\frac{m_{\tilde{W}}}{m_{\tilde{g}}}\right)^{28/19}\left[\left(\frac{m_H}{m_{\tilde{H}}}\right)^{3/19}\left(\frac{m_{\tilde{W}}}{m_{\tilde{H}}}\right)^{4/19}\left(\frac{m_{\tilde{L}}}{m_{\tilde{Q}}}\right)^{3/19}\right]$$

M.C., Pokorski, Wagner

$\downarrow$

strong dependence on $m_{\tilde{H}}, m_{\tilde{W}}, m_{\tilde{g}}$

EXAMPLE:

- in the context of gauge coupling unification

$\Longrightarrow$ common gaugino masses $(M_{1/2})$ at M_{GUT}

$\longrightarrow$ $m_{\tilde{W}}/m_{\tilde{g}} \sim \alpha_2(M_Z)/\alpha_3(M_Z)$

$$T_{SUSY} \simeq m_{\tilde{H}}\left(\frac{\alpha_2(M_Z)}{\alpha_3(M_Z)}\right)^{3/2} \simeq |\mu|/6$$

large T_{SUSY} needs large Higgsino mass

Naturalness $\longrightarrow$ $T_{SUSY} \simeq \mathcal{O}(M_Z)$ if $|\mu| \lesssim 1\,\mathrm{TeV}$

Predictions from Minimal Unification

⟨neglecting GUT/string thresholds⟩

inputs: $1/\alpha_{em}(M_Z)$, $\sin^2\theta_W(M_Z)$, SUSY spectrum

→ unification at $\alpha_G \simeq 0.04$ $M_{GUT} \simeq 2 \cdot 10^{16}$ GeV

$$\Longrightarrow \alpha_3(M_Z) = 0.127 - 4\left(\sin^2\theta_W - 0.2315\right) \pm \underbrace{0.008}_{\text{SUSY SPECTRUM}}$$

Recall:

Experimentally ⟶ $\alpha_3(M_Z) \simeq 0.119 \pm 0.006$

Remarkable agreement between theory & experiment !!

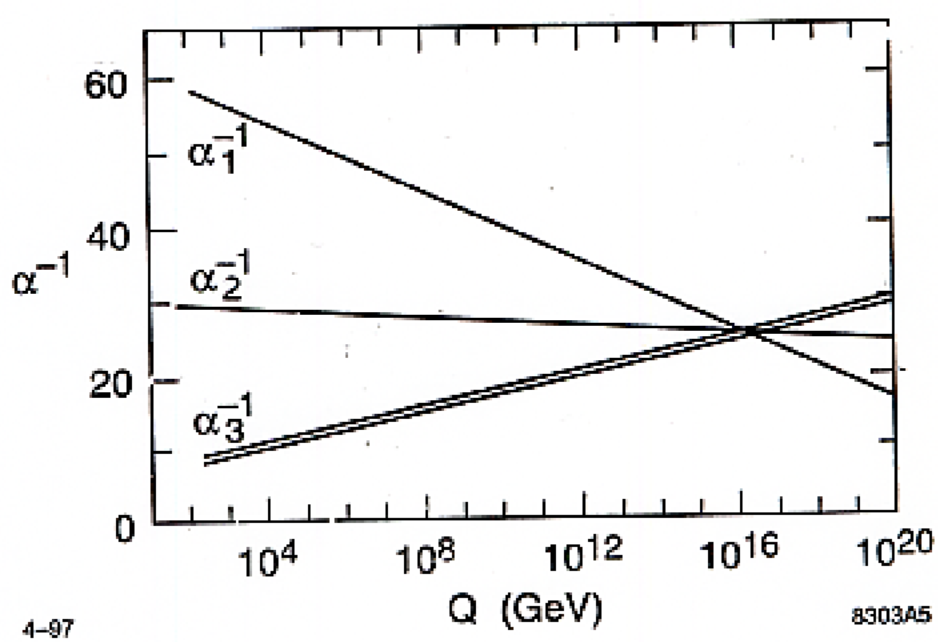
Bardeen, M.C., Pokorski, Wagner
Langacker, Polonsky

T_{SUSY} dep. of $\alpha_3(M_Z)$ predictions

M_t [GeV]	$\sin^2\theta_W(M_Z)$	$T_{SUSY} = 1$ TeV	$T_{SUSY} = M_Z$
150	0.2321	0.116	0.125
→ 170	0.2315	0.118	0.127
190	0.2305	0.122	0.131
		$\alpha_3(M_Z)$	

preferred values: $\alpha_3(M_Z) > 0.116$

Largest uncertainty in $\alpha_3(M_Z)$: from GUT scale physics



60
40
20
0
α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
10^4
10^8
10^{12}
10^{16}
10^{20}
Q (GeV)
4–97
8303A5

- Bottom-Tau Yukawa Coupling Unification

<natural in most Grand Unified models>

⟶ using as additional input

m_b , m_τ and m_t measured values

- within SUSY, two Higgs doublets H_1 , H_2 needed to generate mass to up-& down-quarks and leptons (also to have an anomaly free Higgsino sector)

when $\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$ $\quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$ $\quad \tan\beta \equiv \frac{v_2}{v_1}$

$$\Rightarrow \quad m_t = h_t \, v_2 \qquad m_b = h_b \, v_1 \qquad m_\tau = h_\tau \, v_1$$

with $v_1 = v \cos\beta$; $v_2 = v \sin\beta$; $v = 174$ GeV

Study Yukawa coupling RG evolution

Unification possible?

* clearly important $\tan\beta$ dependence

Yukawa Coupling RG Running:

$$\frac{dY_t}{dt} \simeq \frac{Y_t}{4\pi}\left[\left(16/3\,\alpha_3 + 3\alpha_2 - 6Y_t\right) - Y_b\right]$$

$$\frac{dY_b}{dt} \simeq \frac{Y_b}{4\pi}\left[\left(16/3\,\alpha_3 + 3\alpha_2 - Y_t\right) - 6Y_b\right]$$

$$\frac{dY_\tau}{dt} \simeq \frac{Y_\tau}{4\pi}\, 3\alpha_2$$

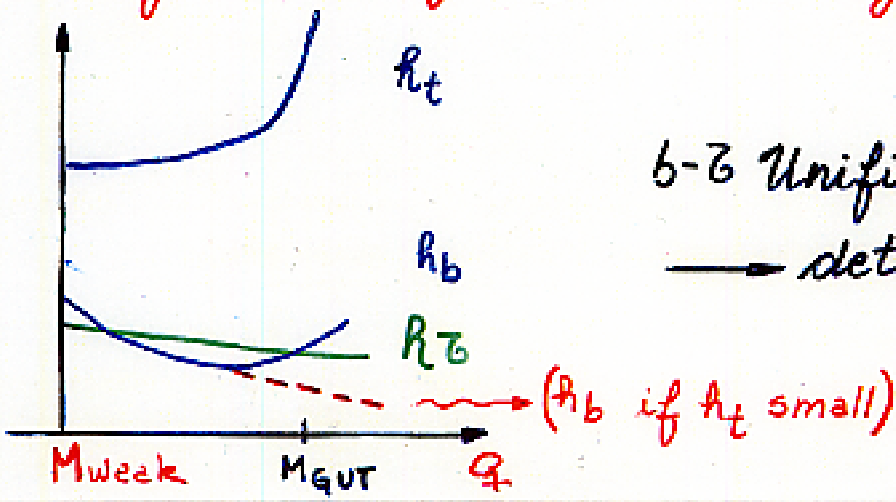
where $Y_i \equiv \frac{h_i^2}{4\pi}$ $\qquad t = \ln\left(\frac{M_{GUT}}{Q}\right)^2$

⟨ low & moderate $\tan\beta \longrightarrow Y_t \gg Y_b, Y_\tau$; α_1 neglected ⟩

to achieve b-τ unification:

strong α_3 renormalization effects on h_b running must be partially suppressed

$\Longrightarrow$ large values of h_t necessary



b-τ Unification $\longrightarrow$ determines h_t

• **Bottom-Tau Yukawa Coupling Unification**

2-loop RG evolution

· given $m_b(M_b)$, $m_\tau(M_\tau)$ and $h_b(M_{GUT}) = h_\tau(M_{GUT})$

$\Longrightarrow$ prediction for h_t

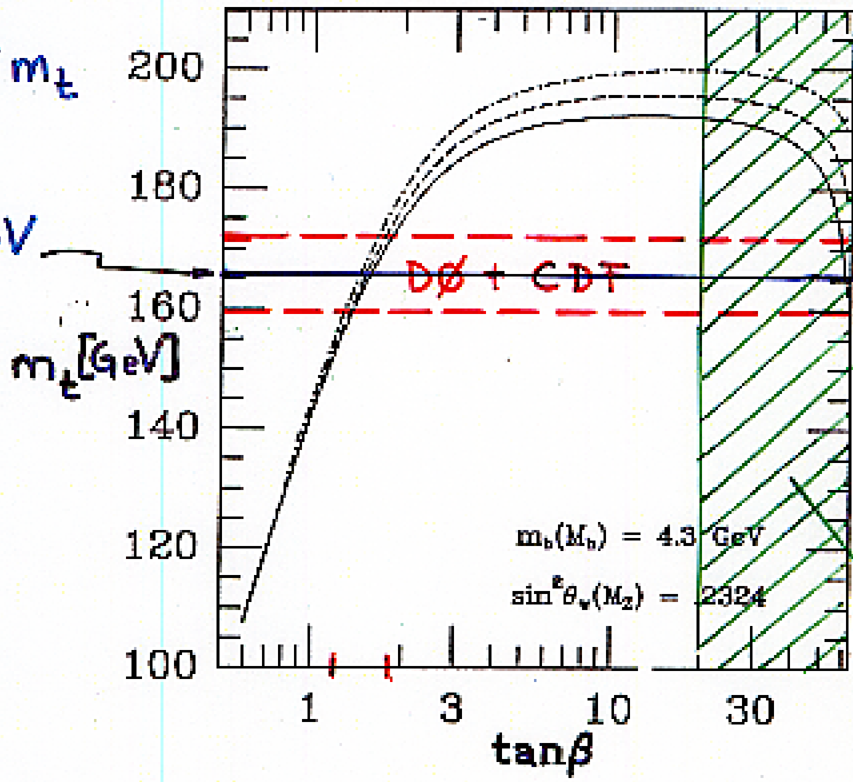
Since $m_t = h_t\, v \sin\beta$ $\Longrightarrow$ prediction for $m_t(\tan\beta)$

$\tan\beta \equiv v_2/v_1$

$\alpha_3(M_Z) =$ —— 0.115, - - - 0.122, -·-·- 0.130

$M_t \simeq 1.05\, m_t$

$M_t = 175$ GeV



Arason, Castaño, Keszthelyi, Mikaelian, Piard, Ramond, Wright '91

Dimopoulos, Hall, Raby '92

Barger, Berger, Ohmann '93

Langacker, Polonsky '93

M.C., Pokorski, Wagner '93

large correc. to m_b need to be included

For $M_t = 175 \pm 6$ GeV $\longrightarrow$ $1 \lesssim \tan\beta \lesssim 2$

$\tan\beta > 30$ $\longrightarrow$ strong spectrum dependence due to large m_b corrections

$\longrightarrow$ b-τ-t unif. possible

• b-τ Unification $\Longleftrightarrow$ large values of $h_t(M_{GUT})$

⟨to compensate α_3 effects in h_b running⟩

At present,

m_h LEP exclusion limit at low $\tan\beta$

$\Rightarrow \tan\beta \lesssim 2.5$ excluded for $M_t = 175$ GeV, $M_{SUSY} \simeq 1$ TeV

$\Longrightarrow$ b-τ unification in low $\tan\beta$ regime strongly disfavoured /(excluded)

but:

Evidence for Neutrino masses & mixings

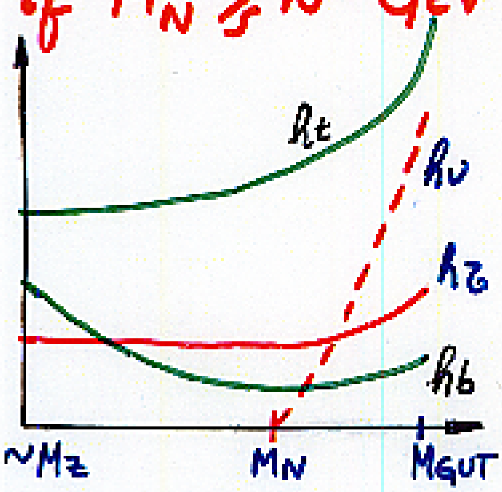
$\Rightarrow$ demands extension beyond SM & open many different possibilities

(SM + $3\nu_R$ → Dirac masses / SM + $3\nu_R$ heavy → Majorana masses /...)

if sol. such that $h_\nu / h_t \ll 1$ ($M_N \lesssim 10^{10}$ GeV or Dirac masses)

$\Rightarrow$ no relevant change in Unification of couplings

if $M_N \gtrsim 10^{14}$ GeV $\Rightarrow$ sizeable h_ν → some effect on m_τ as h_t has on m_b



naively, for low $\tan\beta$ h_ν sizeable renders unif. even more difficult

However

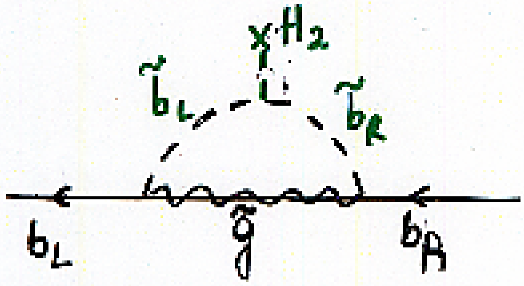
b-τ unification possible for low $\tan\beta \simeq 3$-10 (allowed by LEP) if maximal mixing in lepton sector meas. experimentally is shared by ℓ^+/ν masses at M_{GUT}

At large values of $\tan\beta \Rightarrow h_b$ becomes sizeable and affects its own running in a crucial way

also, important one loop corrections (SUSY loops) to h_b affect the tree level relation

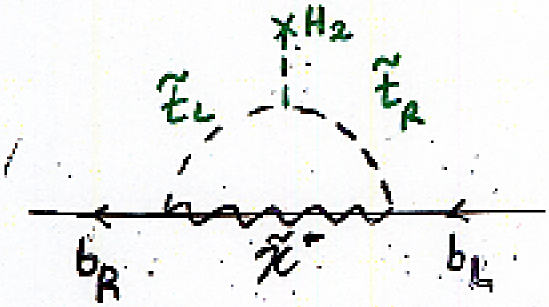
$$m_b = h_b v_1 \longrightarrow m_b = h_b v_1 + \Delta h_b v_2$$

$$= h_b v_1 \left(1 + \Delta h_b / h_b \tan\beta\right)$$



- sizeable correc at large $\tan\beta$
- strong dependence on SUSY spectrum

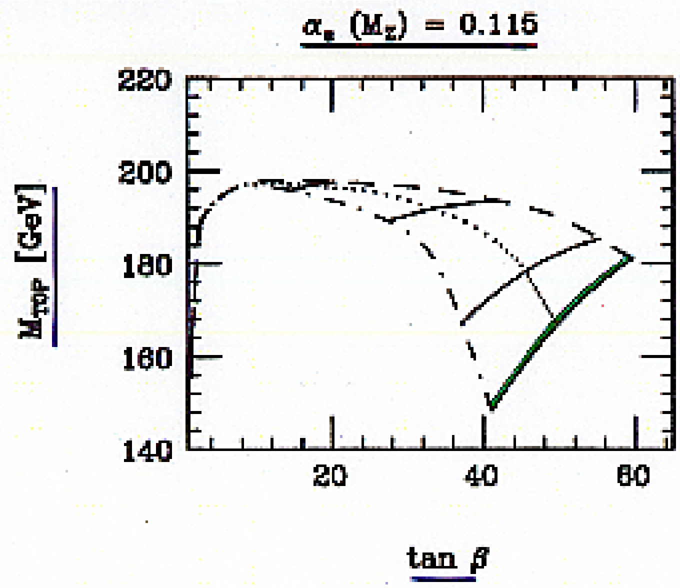
$M_{\tilde{g}}, m_{\tilde{b}_i}, A_t, \mu, m_{\tilde{t}_i}$

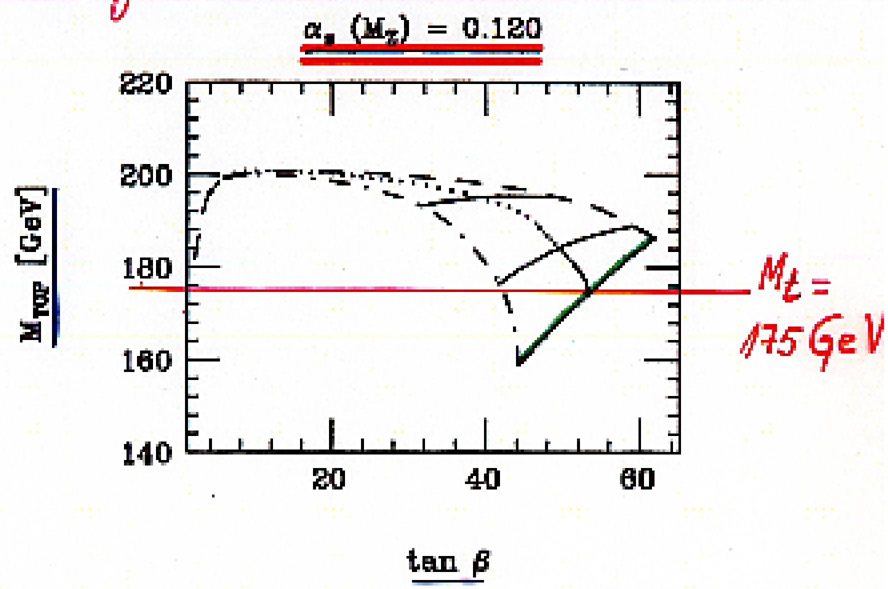


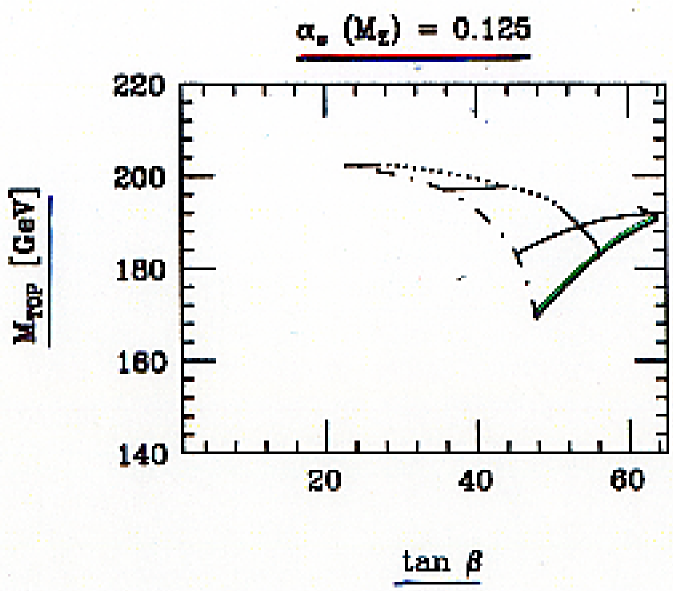
Unification of b-τ Yukawa couplings at M_{GUT} possible depending on $\Delta h_b / h_b \tan\beta$ corrections for given values of $\tan\beta \gtrsim 30$

$(m_b, m_\tau, m_t \rightarrow$ input param$)$

Top Quark Mass Vs. $\tan\beta$: with b-τ Unification for various bottom mass correc.







$\Delta m_b = K \tan\beta$

$K = 0$

$K = -0.003 \rightarrow \simeq 15\%$

$K = -0.006 \rightarrow \simeq 25\%$

$h_t/h_b(M_{GUT}) = 1; 0.6; 0.2; 0.06$

M.C., Dimopoulos, Raby, Wagner '95

The Higgs scalar potential derived from

$$V[H_i] = \left|\partial P/\partial H_i\right|^2 + \frac{1}{2}\sum_a \big(\underbrace{H_i^* T_{ij}^a g_a H_j}_{\equiv D^a}\big)^2 + V_{soft}$$

at tree level

$$\Longrightarrow V_{eff}^{Tree} = m_1^2 H_1^+ H_1 + m_2^2 H_2^+ H_2 - (m_{12}^2\, H_1^T i\tau_2 H_2 + h.c.)$$

$$+\frac{1}{8}(g_1^2+g_2^2)(H_2^+ H_2 - H_1^+ H_1)^2 + \frac{g_2^2}{2}\,|H_2^+ H_1|^2$$

where $m_i^2 = m_{H_i}^2 + |\mu|^2 \longrightarrow \left|\partial P/\partial H_i\right|^2 \rightsquigarrow P[H_i] = \mu\, \varepsilon_{ij} H_1^i H$

$i = 1,2$ soft SUSY term

$m_3^2 = m_{12}^2 \equiv \mu B \longrightarrow$ soft term (bilinear in $P[A]$)

$g_1, g_2 \longrightarrow SU(2), U(1)$ couplings
(g', g)

- quartic couplings {from $\frac{1}{2} D^a D^a$ term only}

$\Longrightarrow$ given as a function of gauge coupling.

Recall:

$$H_1 = \begin{pmatrix} v_1 + \underbrace{\phi_1^0}_{\Rightarrow H_1^0} + iA_1 \\ \phi_1^- \end{pmatrix}; \qquad H_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \phi_2^0 + iA_2 \end{pmatrix} e^{i\xi}; \quad \text{and}$$

$$* \ (H_1^+ H_1)(H_2^+ H_2) = |H_2^+ H_1|^2 + |H_2^+ i\tau_2 H_1^*|^2 \qquad \tan\beta \equiv v_2/v_1$$

Tree-level mass predictions

- **CP-odd sector**

$$V_{eff.} \longrightarrow (A_1 \; A_2) \begin{bmatrix} m_1^2 - M_Z^2/2 \cos 2\beta & m_3^2 \\ m_3^2 & m_2^2 - M_Z^2/2 \cos 2\beta \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

using minimization conditions, $\partial V / \partial H_i \big|_{\langle H_i \rangle = v_i} = 0$

$$v_2^2 / v_1^2 \equiv \tan^2\beta = \frac{m_1^2 + M_Z^2/2}{m_2^2 + M_Z^2/2}$$

and

$$\sin 2\beta = 2\, m_3^2 / (m_1^2 + m_2^2)$$

$\Longrightarrow$ det $M_A^2 = 0$ $\qquad$ $Tr[M^2] = m_1^2 + m_2^2$

$\longrightarrow$ one CP-odd Goldstone-boson, $m_G = 0$

and CP-odd Higgs boson A

$$\underline{\underline{m_A^2 = m_1^2 + m_2^2}} \equiv m_{H_1}^2 + m_{H_2}^2 + 2\mu^2$$

- **Charged sector**

$\longrightarrow$ (similar) det $M_{H^\pm}^2 = 0 \longrightarrow$ Goldstone-boson

and $Tr[M_{H^\pm}^2] \equiv \underline{\underline{m_{H^\pm}^2 = m_A^2 + M_W^2}}$

$\Longrightarrow$ charged Higgs boson $H^\pm$

• CP- even sector

$$V_{eff} \to (H_1^\circ \; H_2^\circ) \begin{pmatrix} M_Z^2 \cos^2\beta + m_A^2 \sin^2\beta & -(M_Z^2+m_A^2)\sin\beta\cos\beta \\ -(M_Z^2+m_A^2)\sin\beta\cos\beta & M_Z^2\sin^2\beta + m_A^2\cos^2\beta \end{pmatrix} \begin{pmatrix} H_1^\circ \\ H_2^\circ \end{pmatrix}$$

$\Longrightarrow$ 2 CP-even mass eigenstates h, H

$$h = \cos\alpha\, H_2^\circ - \sin\alpha\, H_1^\circ \qquad H = \cos\alpha\, H_1^\circ + \sin\alpha\, H_2^\circ$$

$$\Longrightarrow \quad m^2_{H,h} = \frac{m_A^2 + M_Z^2}{2} \pm \frac{\sqrt{(m_A^2+M_Z^2)^2 - 4M_Z^2 m_A^2 \cos^2 2\beta}}{2}$$

all masses given as a function of

tan β , m_A and gauge couplings

• if $m_A^2 \gg M_Z^2$

$$\Longrightarrow \quad m_h = M_Z\,|\cos 2\beta| \qquad \text{hence}$$

$$m_h^{max} \leq M_Z \qquad \text{(tree level !)}$$

Minimal Supersymmetric theories predict a light Higgs

while $m_H = m_A$

Higgs effective potential:

most general $\rightarrow$ expansion up to dim. 4 operators

$$
\begin{aligned}
V_{eff} = {} & m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_3^2 \left(H_1^T i \tau_2 H_2 + h.c.\right) \\
& + \frac{\lambda_1}{2}\left(H_1^\dagger H_1\right)^2 + \frac{\lambda_2}{2}\left(H_2^\dagger H_2\right)^2 + \lambda_3 \left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4 \left|H_2^\dagger i \tau_2 H_1^*\right|^2 \\
& + \left\{ \frac{\lambda_5}{2}\left(H_1^T i \tau_2 H_2\right)^2 + \left[\lambda_6 H_1^\dagger H_1 + \lambda_7 H_2^\dagger H_2\right]\left(H_1^T i \tau_2 H_2\right) + h.c. \right\}
\end{aligned}
$$

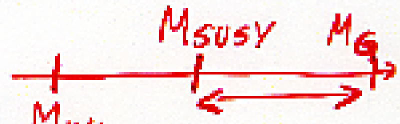
quartic couplings

tree level values $\Longleftrightarrow$ SUSY limit

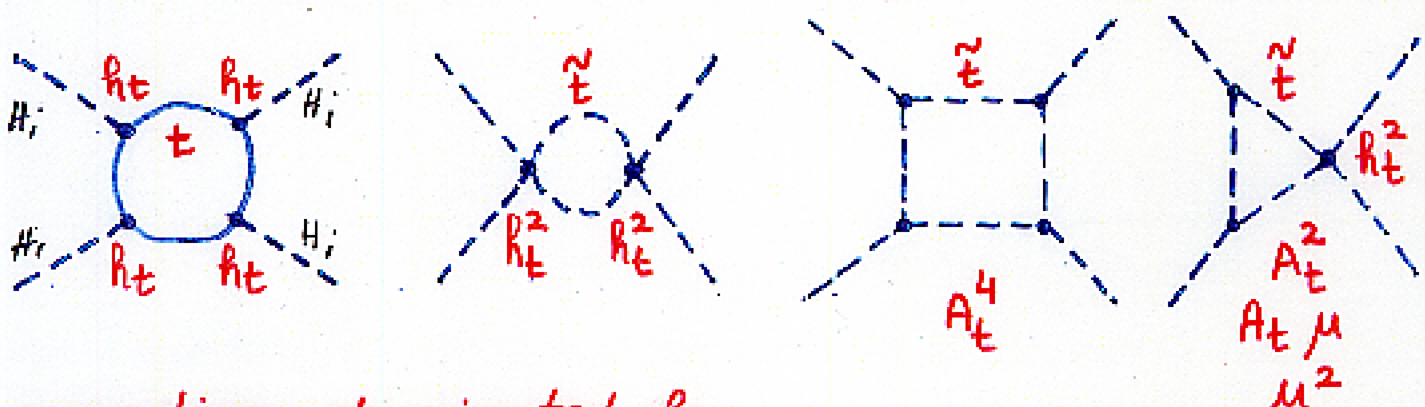
$$\lambda_1 = \lambda_2 = \left(g_1^2 + g_2^2\right)/4 \qquad \lambda_3 = \left(g_2^2 - g_1^2\right)/4$$

$$\lambda_4 = -g_2^2/2 \qquad \lambda_{5,6,7} = 0$$

below/at the scale of SUSY breaking



$\longrightarrow \lambda_i$ get renormalized beyond the gauge coupl.



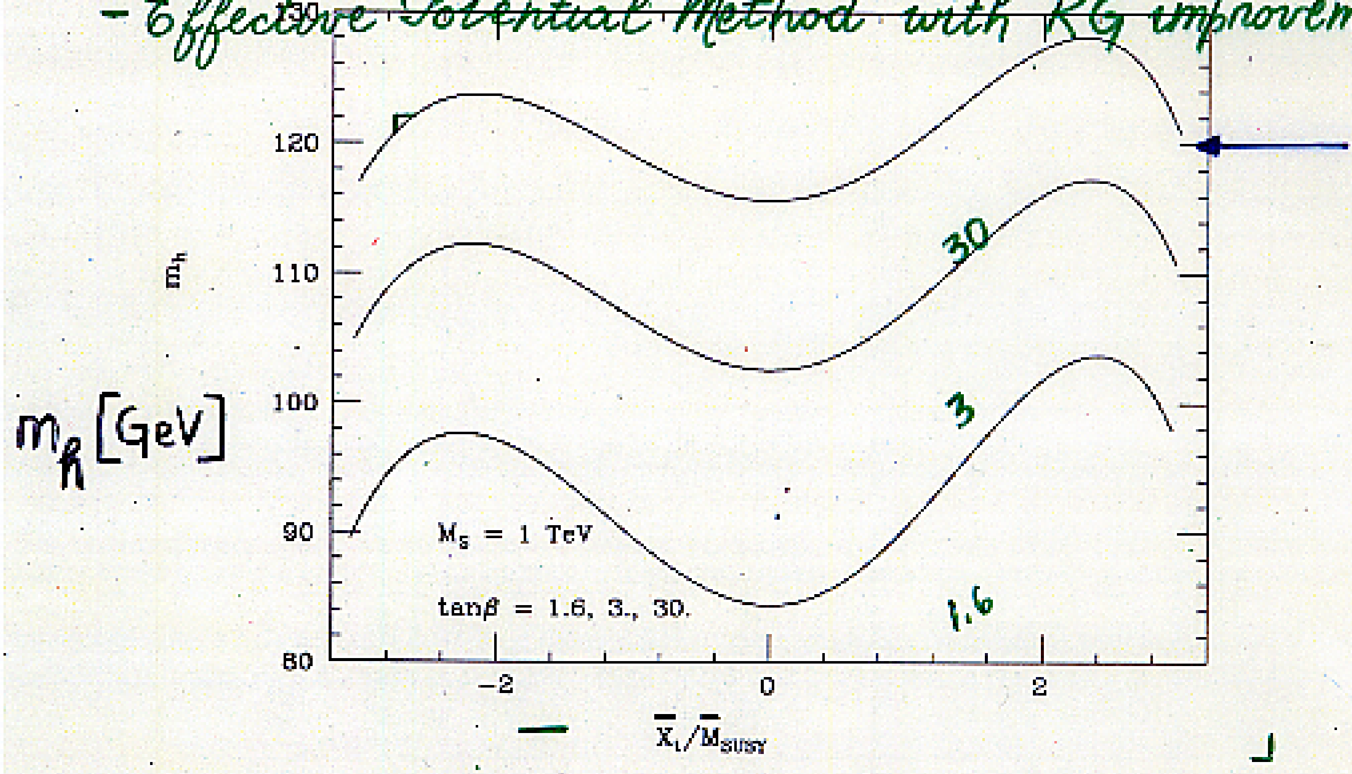
corrections dominated by

top and stop effects (sbottoms also if large $\tan\beta$)

Lightest Higgs mass as a fc. of the stop mixing

latest developments on 2-loop. rad. correc.

- Feynman diagrammatic approach
- Effective Potential Method with RG improvement



$=[A_t - \mu/\tan\beta]/M_{SUSY}$

for

$m_A = M_{SUSY} = 1$ TeV

$M^2_{SUSY} = (m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})/2$

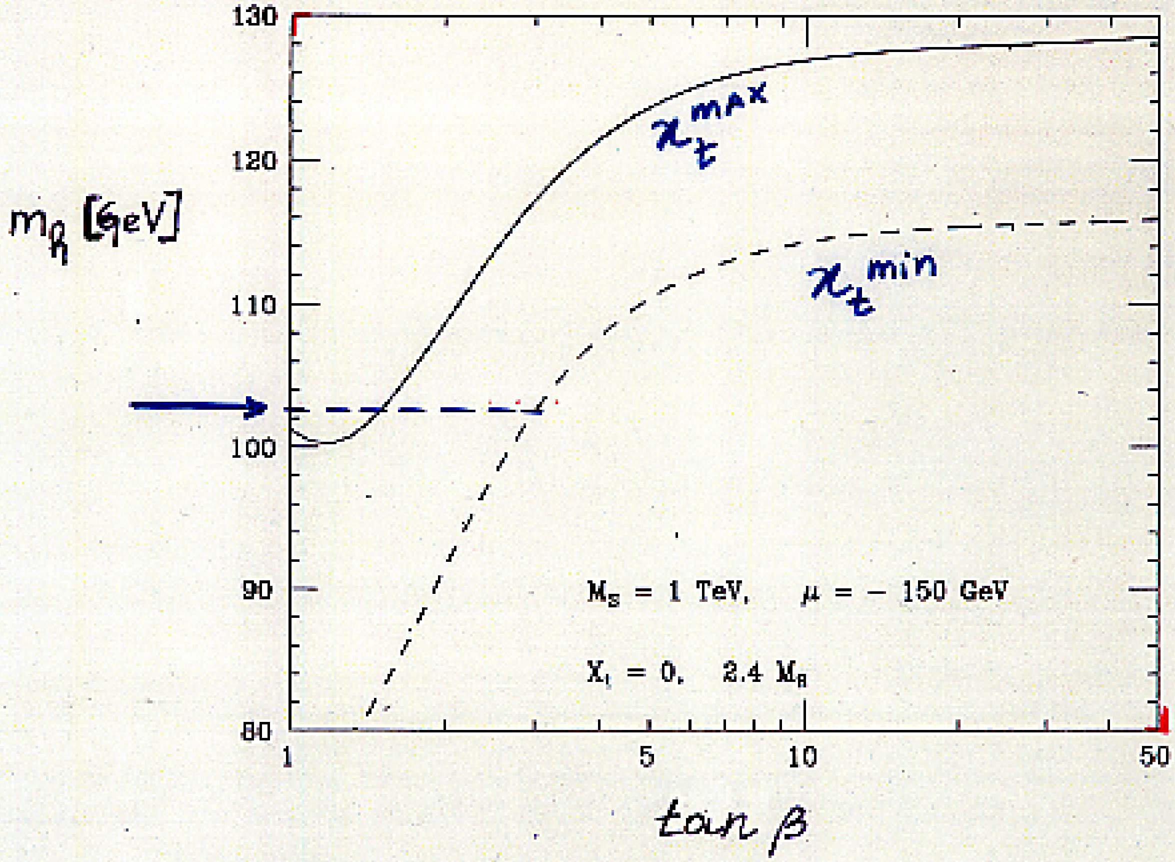
$m_t = 175$ GeV

$m_h^{MAX} < 130$ GeV

1

Carena, Haber, Heinemeyer, Hollik, Weiglein, Wagner '00

Upper bound on m_h as a fc. of $\tan\beta$



large $m_A = 1\,TeV$

$M_{SUSY} = 1\,TeV$

with max. effect from stop mixing sector

$$X_t^{MAX} = A_t - \mu/\tan\beta \simeq \sqrt{6}\, M_{SUSY}$$

low $\tan\beta \to$ LEP limit $m_h \lesssim 113$ GeV excluded

$\Rightarrow \tan\beta < 2.5$ excluded

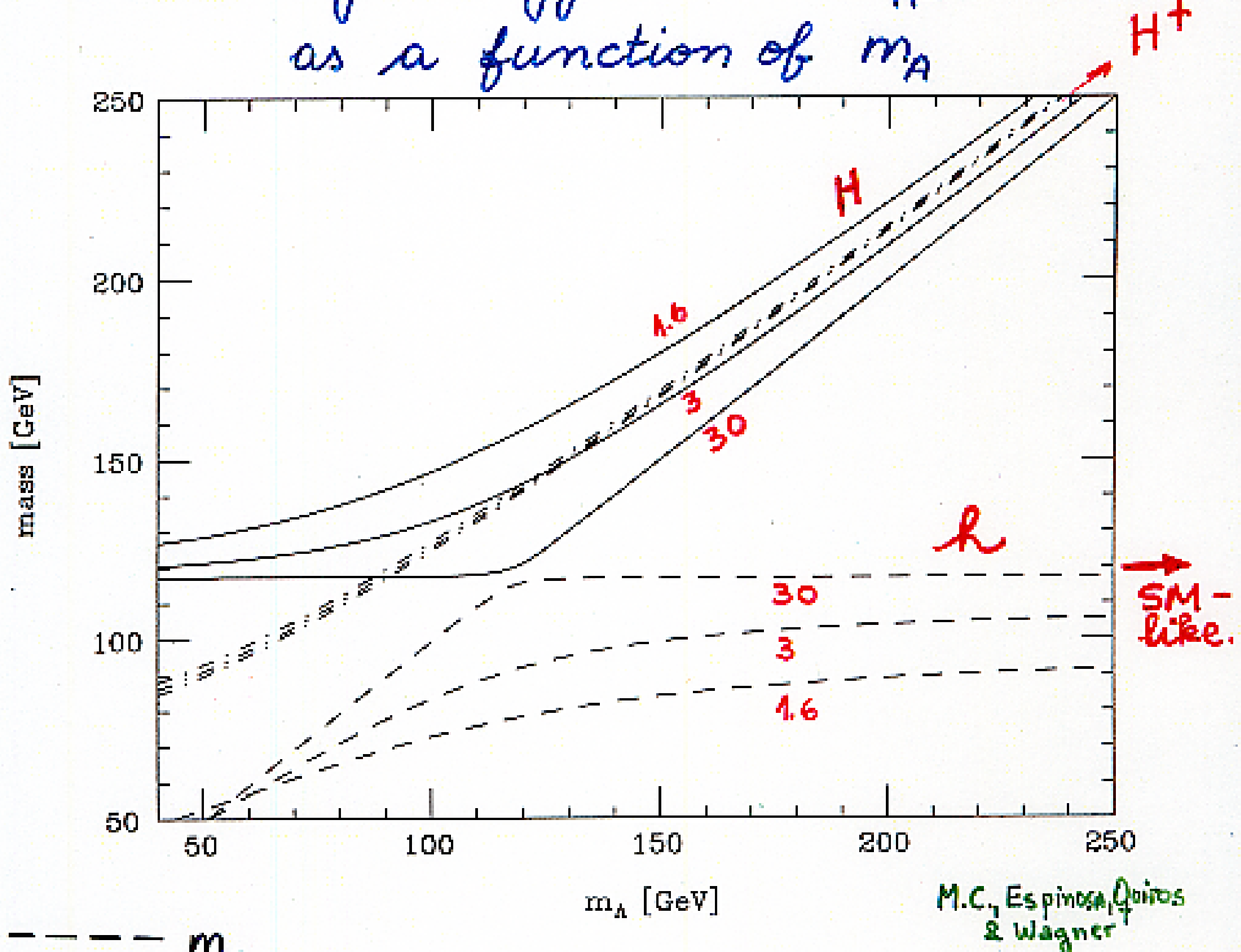
ncertainties:

$\Delta M_{SUSY} = 1 \to 2\,TeV \;\Rightarrow\; \Delta m_h \simeq (2-5)\,GeV$ (mix. dep.)

$\Delta m_t = 1\,GeV \Rightarrow \Delta m_h \sim 1\,GeV$

5

CP-even neutral Higgs Masses $m_{h,H}$ and charged Higgs Mass $m_{H^\pm}$ as a function of m_A



- - - - m_h

——— m_H

-·-·- $m_{H^\pm}$

for $\tan\beta = 1.6, 3, 30$

$M_t = 175$ GeV

$M_{SUSY} = 1$ TeV

$A = -\mu = M_{SUSY}$

(if $m_A < M_{SUSY}$ $\Downarrow$ 2 Higgs doublets in the spectrum below M_S)

for large $\tan\beta \longrightarrow m_H^2$ or m_h^2 given by

(SM-like VV coupl.!) $\quad M_Z^2 +$ rad. correc. $\lesssim 130$ GeV

Neutral Higgs Boson Couplings to Fermions and Gauge Bosons

$$g_{hVV} = g_V m_V \sin(\beta-\alpha)$$

$$g_{HVV} = g_V m_V \cos(\beta-\alpha)$$

$V = W, Z$

$$g_V = \begin{cases} g & (V=W) \\ g/\cos\theta_W & (V=Z) \end{cases}$$

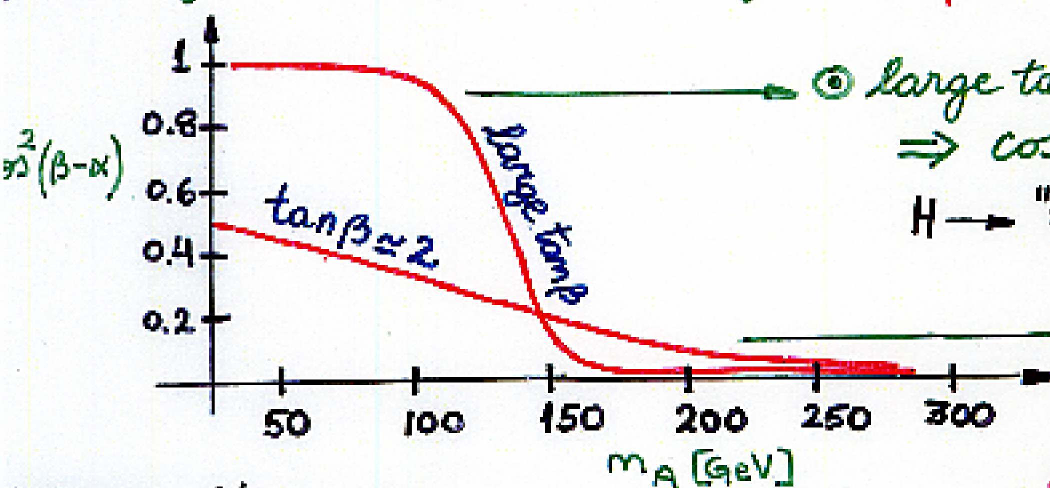
$$g_{hAZ} = \frac{g}{2\cos\theta_W}\cos(\beta-\alpha)$$

$$g_{HAZ} = \frac{-g}{2\cos\theta_W}\sin(\beta-\alpha)$$

(enhanced for large $\tan\beta$)

$g_{ht\bar{t}} = \frac{m_t}{v}\cos\alpha/\sin\beta$ $\qquad$ $g_{hb\bar{b}} = -\frac{m_b}{v}\sin\alpha/\cos\beta$ $\qquad$ $g_{Ab\bar{b}} \propto \tan\beta$

$g_{Ht\bar{t}} = \frac{m_t}{v}\sin\alpha/\sin\beta$ $\qquad$ $g_{Hb\bar{b}} = \frac{m_b}{v}\cos\alpha/\cos\beta$ $\qquad$ $g_{At\bar{t}} \propto \cot\beta$



⊙ large $\tan\beta$, low m_A
$\Rightarrow \cos(\beta-\alpha) \to 1$
$H \to$ "SM-like"

⊙ large m_A
$\sin(\beta-\alpha) \to 1$
$h \to$ SM-like
DECOUPLING LIMIT

define $\langle\phi_W\rangle = v$

$$\phi_W = h\sin(\beta-\alpha) + H\cos(\beta-\alpha)$$

couples to W, Z in SM way $\Longrightarrow$ EW ~~symm.~~

Also

$$m_H^2\cos^2(\beta-\alpha) + m_h^2\sin^2(\beta-\alpha) = m_h^{2\,MAX} \lesssim 130 \text{ GeV}$$

$\Rightarrow$ for large $\tan\beta$ always one CP-even Higgs with SM-like coupl. to W, Z and mass $\lesssim$ 130 GeV

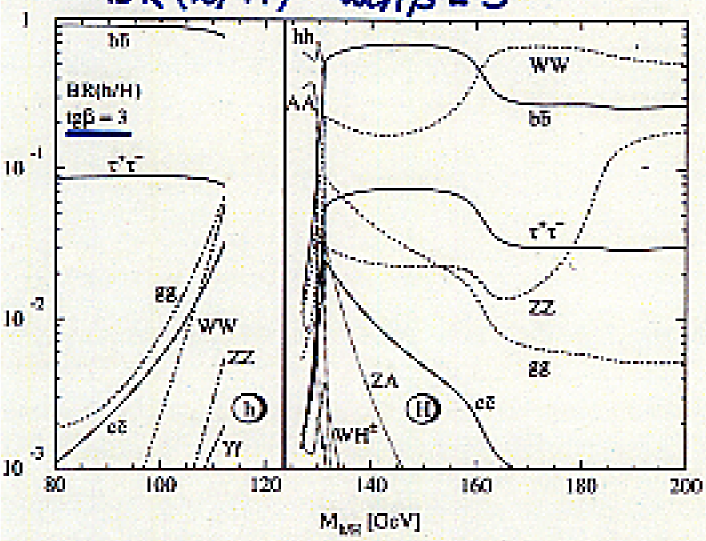
Decay Patterns of MSSM Higgs Bosons

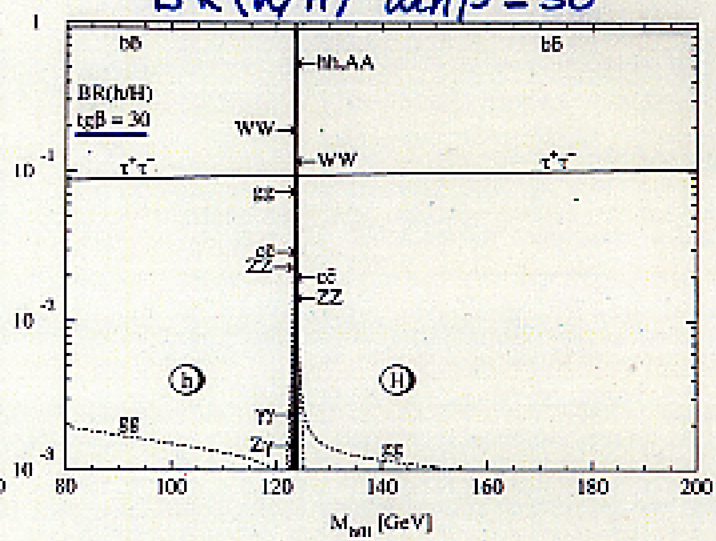
large $\tan\beta$ → strong enhancement to down-type ferm

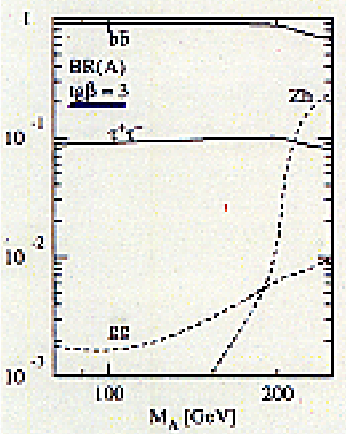
low $\tan\beta$ → richer pattern

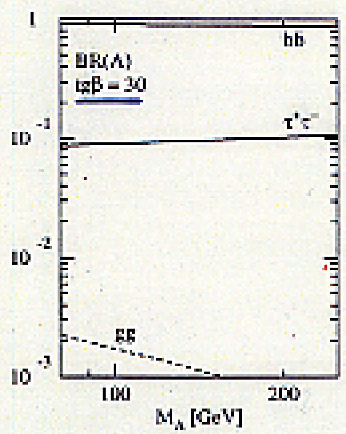
(still h, H, A primarily to $b\bar{b}$, $\tau^+\tau^-$ if $m_\phi \lesssim 150$ GeV)

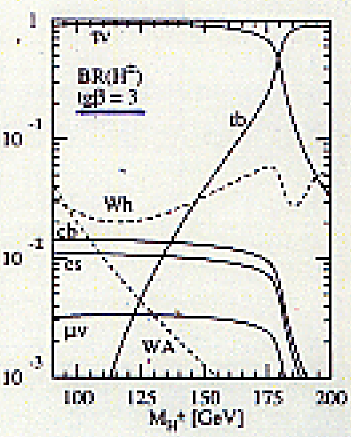
BR(h/H) $\tan\beta = 3$ BR(h/H) $\tan\beta = 30$

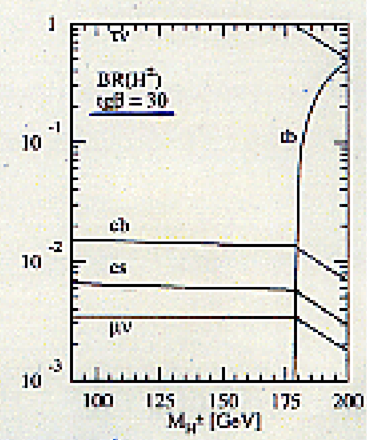












BR(A) BR(H±)

$M_{SUSY} = 1$ TeV $X_t = \sqrt{6}\, M_{SUSY}$

Decay into SUSY particles → when open very important

Radiative corrections to the Higgs Boson Couplings to Fermions & Gauge Bosons

1) Through the radiative corrections to the CP-even Higgs mass matrix in the H_1^0, H_2^0 base

$$\mathcal{M}^2 = \begin{bmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{21}^2 \\ \mathcal{M}_{12}^2 & \mathcal{M}_{22}^2 \end{bmatrix}$$ which defines the

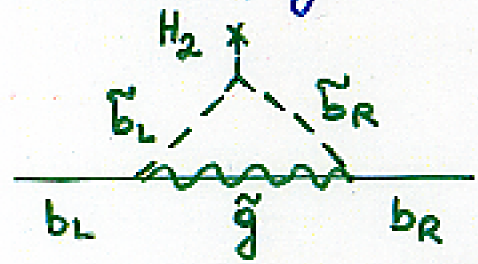
<u>mixing angle α</u>

$$\sin\alpha \cos\alpha = \mathcal{M}_{12}^2 / \sqrt{\mathrm{Tr}(\mathcal{M}^2)^2 - 4 \det \mathcal{M}^2}$$

Important effects of rad. correc. on $\sin\alpha$ and $\cos\alpha$ depending on sign of μA_t and magnitude of A_t/M_{SUSY}

2) SUSY radiative corrections to bottom Yukawa coupling

$$\mathcal{L} \to h_b H_1^0 b\bar{b} + \Delta h_b H_2^0 bb \longrightarrow$$



$\Delta h_b \neq$ even if $M_{SUSY} \to \infty$ and it modifies the $m_b - h_b$ relation

$$m_b = h_b v_1 + \Delta h_b v_2 = h_b v_1 \left(1 + \overbrace{\frac{\Delta h_b}{h_b} \tan\beta}^{\equiv \Delta m_b}\right)$$

for large $\tan\beta \Rightarrow \Delta m_b \simeq \mathcal{O}(1)$!

Modified Higgs Boson Couplings to b-quarks

$$g_{h b\bar{b}} \simeq \frac{-\sin\alpha \; m_b}{v\cos\beta\,(1+\Delta m_b)}\left(1 - \Delta m_b / \tan\alpha\tan\beta\right)$$

$$g_{H b\bar{b}} \simeq \frac{\cos\alpha \; m_b}{v\cos\beta\,(1+\Delta m_b)}\left(1 + \Delta m_b \tan\alpha / \tan\beta\right)$$

$$g_{A b\bar{b}} \simeq h_b \sin\beta = \frac{m_b}{v\,(1+\Delta m_b)} \tan\beta$$

M.C., Mrenna, Wagner

important resummation
included through EFT approach

M.C, Garcia, Nierste, Wagner

Important modifications to $g_{h(H) b\bar{b}}$ may occur for special regions of MSSM param. space

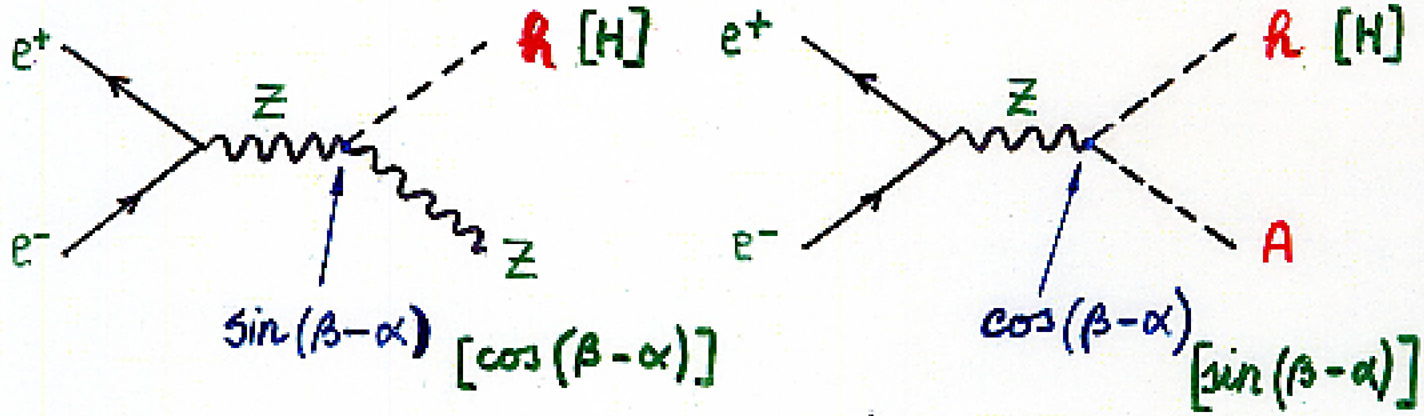
<dep. on sign and values of μA_t, μA_b, $\mu M_{\tilde{g}}$ and magnitude of $M_{\tilde{g}}/M_{SUSY}$, μ/M_{SUSY}>

$$\Delta m_b \equiv \Delta h_b/h_b \tan\beta \propto \frac{2\alpha_3}{3\pi} M_{\tilde{g}}\,\mu\, I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, M^2_{\tilde{g}}) \tan\beta + h_t^2/16\pi^2 \, A_t\,\mu\, I(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}, \mu^2) \tan\beta$$

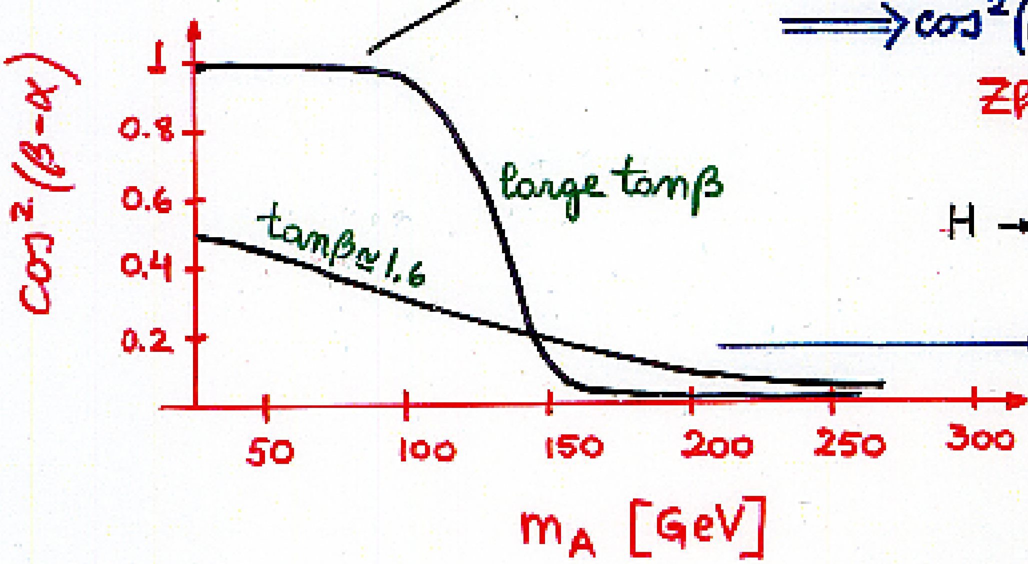
* Strong suppression of $g_{h(H) b\bar{b}}$ couplings if $\tan\alpha \simeq \Delta m_b/\tan\beta$ $\left(\tan\alpha^{-1} \simeq -\Delta m_b/\tan\beta\right)$
  $\Longleftrightarrow$ $\sin\alpha$ or $\cos\alpha$ very small

MSSM Higgs Boson Production at LEP

Higgs strahlung • Associated pair production



$$\left.\begin{aligned}\sigma(e^+e^- \to Zh) &= \sin^2(\beta-\alpha)\,\sigma_{SM}\\ \sigma(e^+e^- \to hA) &= \cos^2(\beta-\alpha)\,\bar{\lambda}\,\sigma_{SM}\end{aligned}\right\} \Longrightarrow \text{complementary}$$



large $\tan\beta$ + low m_A

$\Longrightarrow \cos^2(\beta-\alpha) \longrightarrow 1$

ZhA channel enhanced

H → SM-like coupl.

large m_A

$\Downarrow$

$\sin^2(\beta-\alpha) \to 1$

h → SM like

- WW fusion process → also $\sin^2(\beta-\alpha)$ factor
  marginal but useful at the kin. limit of Zh

MSSM Higgs Search at LEP

1) $e^+e^- \longrightarrow Zh$ $\quad \sigma(e^+e^- \rightarrow Zh) = \sin^2(\beta-\alpha)\, \sigma(e^+e^- \rightarrow Z H_{SM})$

$h \rightarrow b\bar{b} / \bar{\tau}\tau$

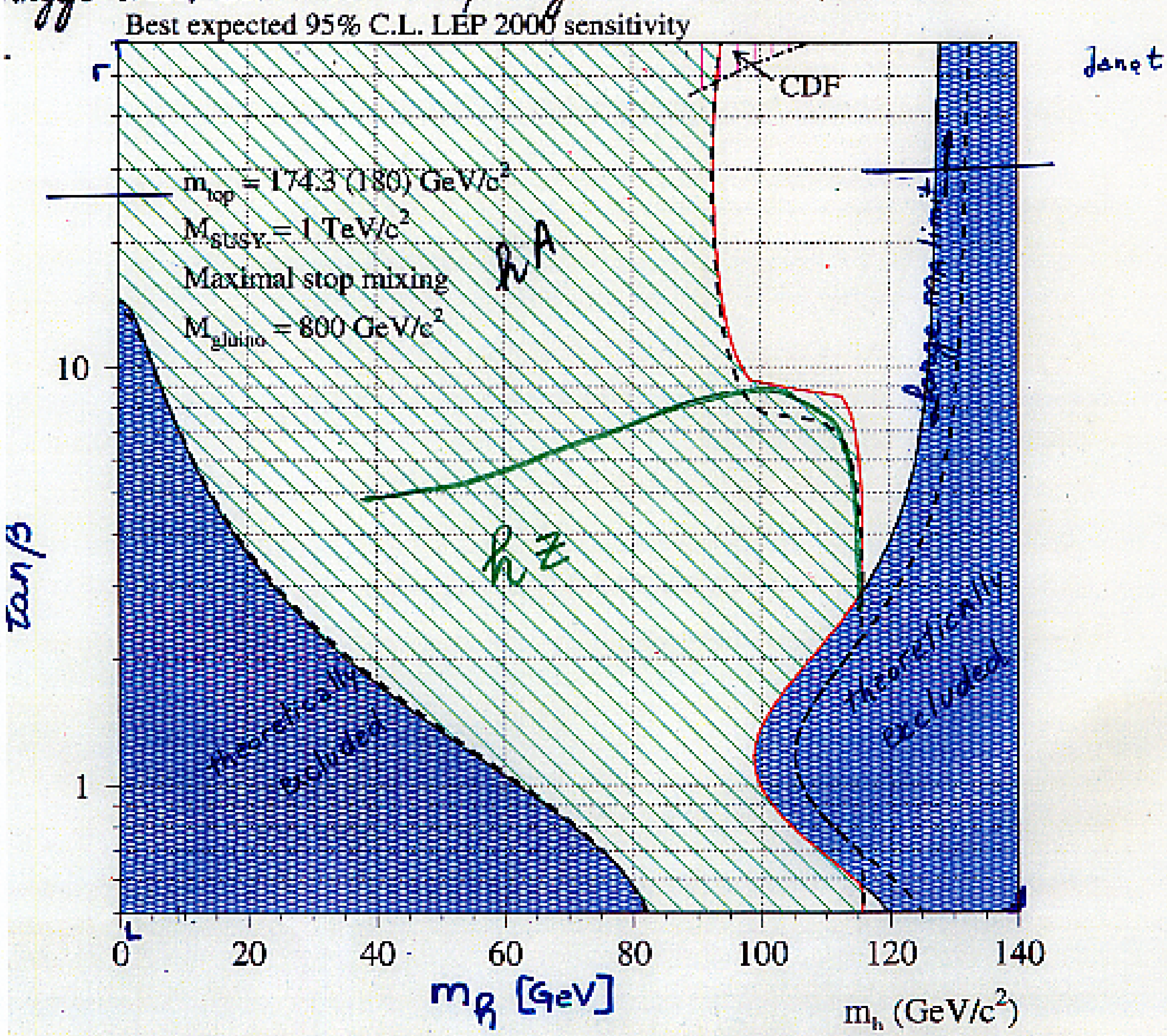
good channel @ large m_A
low $\tan\beta$ + low m_A

2) $e^+e^- \longrightarrow hA$ $\quad \sigma(e^+e^- \rightarrow hA) \propto \cos^2(\beta-\alpha)\, \sigma(e^+e^- \rightarrow Z H_{SM})$

$hA \rightarrow b\bar{b}b\bar{b} / \tau^+\tau^- b\bar{b}$

good channel @ large $\tan\beta$, low m_A

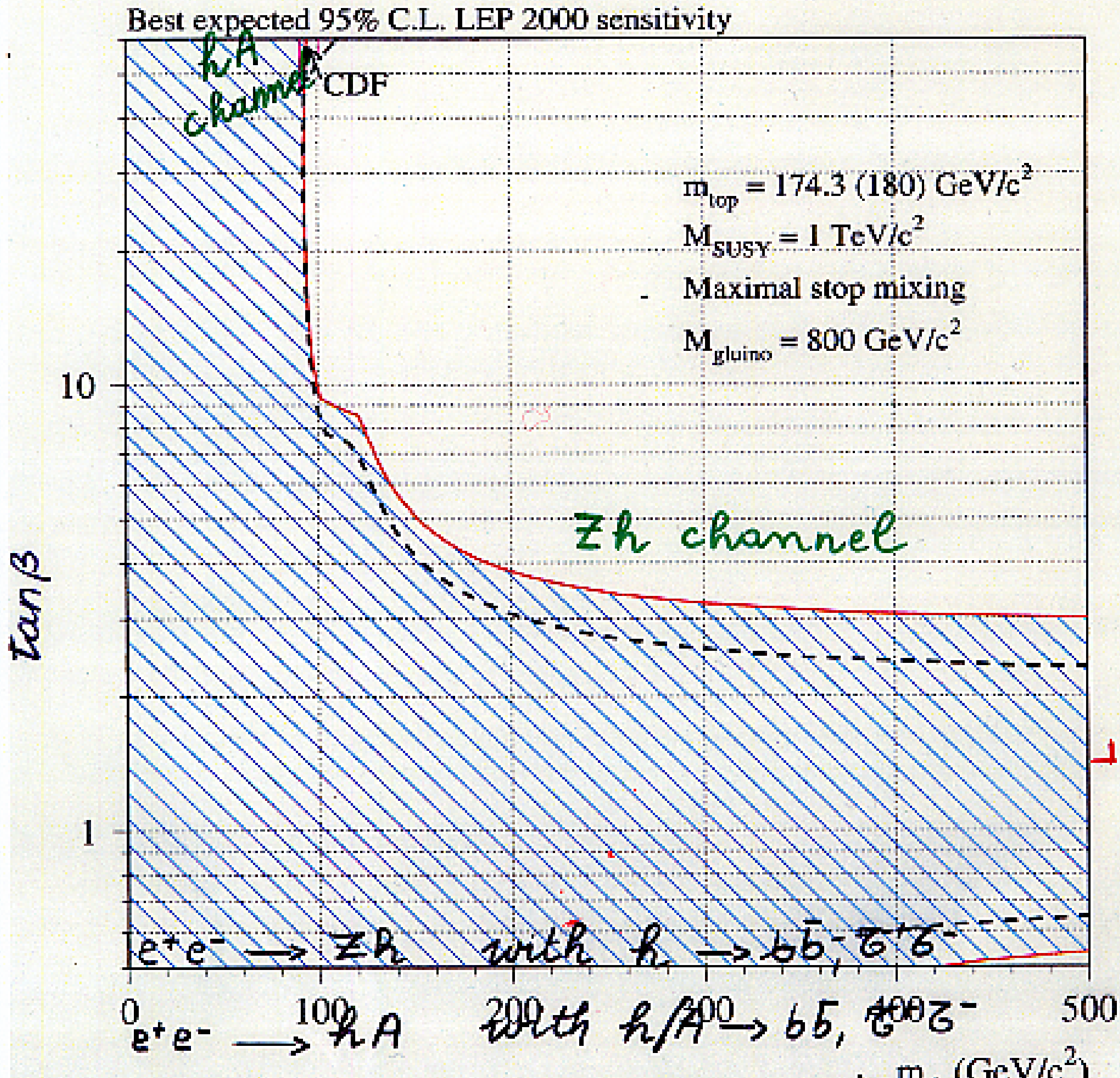
Complementary channels, but only (1) searches for ϕ_W Higgs with SM-like couplings to vector bosons



Benchmark → in general, the most difficult region
but some exceptions

LEP2 reach in the $\tan\beta - m_A$ plane

≃ SM limit



(also marginally ZH, HA)